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Mixing by Resonant Acoustic Driving in a Closed Chamber

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In this study the effects of resonant acoustic oscillations on gas phase mixing in a rectangular cavity without mean flow are investigated. The primary purpose of this research is to provide insight into how acoustic amplitude, acoustic mode, and acoustic streaming influence the gas phase mixing rate inside the cavity. Experimental results show significant enhancement of mixing rate due to the presence of acoustics in the mixing chamber. Increased acoustic pressure amplitudes in the chamber were shown to increase the mixing rate. The dependence of mixing rate upon the mode of acoustic excitation was found to be highly complex. Acoustic streaming observed and measured in the cavity appears to have a significant influence upon mixing rate.

Nomenclature

I_{av} = average pixel intensity of an image, $(1/m) \Sigma_{n=1}^{m} I_n$
I_n = intensity of the nth pixel of an image
m = number of pixels/frame
p = acoustic pressure amplitude
t = time after sound initiation
Us = spatial unmixedness parameter
$\tau(\theta)$ = time for Us to decrease to θ

Introduction

THIS work is part of a study to determine whether the application of acoustic oscillations can improve the efficiency and reduce the size of incinerators. Rapid mixing is essential to efficient combustion in propulsion and power generation systems as well as in the design of compact, efficient combustors with low emissions. While considerable effort has been expended on controlling and eliminating unwanted and often harmful oscillations in combustion systems, research conducted in recent years has demonstrated that flow pulsations and acoustic oscillations can increase the rates of transport processes. Patera et al.[1,2] have shown that mixing and heat transfer rates in laminar flow in a channel can be considerably increased by oscillating the flow rate into the channel at frequencies at which the shear layer of the flow is unstable. While there is considerable evidence showing that mass[3] and heat[4–6] transfer rates are increased by acoustic oscillations, the precise mechanism responsible for these increases is not entirely understood. Strong evidence[4,7] suggests that the increased transport rates are due to the excitation of turbulence and vortical structures by the acoustic oscillations. In a study by Vermulen et al.,[8] flow oscillations were shown to reduce flow stratification and improve mixing, as evidenced by the elimination of hot spots in the temperature profile at the exit plane of a gas turbine combustor.

To better understand the mechanisms of mixing rate enhancement due to acoustic oscillations isolated from the mixing effects of the mean flow in and out of the combustor, the characteristics of gas phase mixing in an environment of large-amplitude, resonant, acoustic oscillations in a rectangular cavity with no mean flow was invested. Previous research has shown[9] that under such conditions, acoustic streaming often plays a significant role in the mixing process. It is well known that periodic sound sources can generate nonperiodic motions of the medium.[10–12] Eckart,[10] in his paper on streaming and vorticity generated by sound waves, shows that circulation necessarily follows as part of the solution to the wave equation when one accounts for the viscosity and second-order terms. While such motions could provide effective means for enhancing mixing processes, their effect upon mixing rates in cavities has not been investigated to date. This article reports the results of an experimental study of the effects of the mode and amplitude of resonant acoustic oscillations in a cavity upon the rate of gas phase mixing.

Experimental Facility

A schematic of the experimental setup used in the mixing rate studies is shown in Fig. 1. It consists of a rectangular wooden box with one acrylic side. The cavity has dimensions of 68.6 × 20.3 × 12.7 cm. These dimensions were chosen to prevent redundant natural frequencies over the range of interest. A removable partition can be inserted into the middle of the box to divide the cavity into two sections. Four University Sound, 75-W, compression type, acoustic drivers were used to excite standing acoustic waves in the box. The drivers were mounted on the ends and bottom of the setup, which allowed longitudinal, transverse, and multidimensional acoustic modes to be excited. Sound pressure levels as high as 158 dB can be driven for certain modes.

In this article, the natural acoustic modes are identified by the longitudinal mode number (in the 68.6 cm direction) and the mode number in the 20.3 cm direction (referred to as the transverse mode). Oscillations in the 12.7 cm direction were

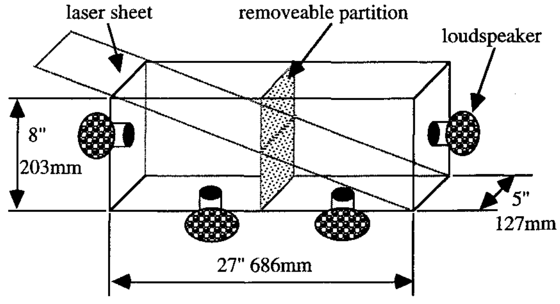

Fig. 1 Schematic of the experimental setup.

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not excited in this study, and so the mode numbers in this direction are assumed to be zero. For example, by this convention, the (1,0) mode refers to the fundamental longitudinal mode and the (2,0) mode refers to the second longitudinal mode, the (1,1) mode and the (3,1) mode refer to the multidimensional modes that are combinations of the first longitudinal and the first transverse modes and the third longitudinal and the first transverse modes, respectively.

The acoustic driving in the setup can be achieved by either open- or closed-loop operation. In open-loop operation, a signal to the drivers is provided by a function generator at a chosen frequency. In closed-loop active control operation, the acoustic pressure signal in the chamber, measured using a piezoelectric pressure transducer, is amplified, phase-shifted, and fed back into the acoustic drivers. By adjusting the gain, phase-shift, position of the pressure probe, and the combination of utilized drivers, the system can be tuned to resonate in various acoustic modes. Whereas open-loop operation is advantageous in its simplicity, closed-loop active control provides the benefit of automatically finding and locking on to various resonant modes.

The removable partition in the middle of the cavity, shown in Fig. 1, was used to establish an initial condition for the experiment. Before each test, the partition was inserted and half of the box was filled with smoke (i.e., one side of the partition). The smoke-filled air in this half of the box was well-mixed and allowed to become quiescent. The rate of mixing between smoke-filled air and smoke-free air in the cavity was measured under various modes of acoustic excitation. A high-speed, intensified, digital imaging system was used to record and quantify the mixing process on a plane illuminated by a light sheet (see Fig. 1). The light sheet was created with an argon ion laser and the necessary optics, and a Kodak EctaPro intensified charge-coupled device camera was used to acquire images of the Mie scattering from the smoke particles. At the beginning of each test, the partition was then removed, the camera was triggered, and oscillations were excited in the box. The camera, which has a maximum resolution of 238 × 191 pixels and 3634 frame memory buffer, was configured to record 50 frames/s with a 20-μs gate time that effectively froze the motion. This allowed a maximum run time of 72 s. The collected images were then downloaded to an Intel 486 based computer for storage and analysis.

Analysis Method

A sequence of frames from a typical test is shown in Fig. 2. The first frame shows the initially separate smoke and air in the box just after the partition has been removed and as the resonant acoustic driving is initiated. As time increases, mixing proceeds from large to small scale until, as seen in the final frame, mixing is complete to the scale of the resolution of the camera. Comparison of the relative degree of mixing between two frames can be obtained from the histograms of the images, which show the total number of pixels in an image at each of the possible discrete intensity levels. The black and white intensified camera used in this study has eight bits of gray scale resolution, which result in 256 possible discrete intensity levels for each pixel. Figure 3 shows the histograms of the frames as they evolve in time. At the beginning of a test, the histogram of the pixels with respect to light intensity is bimodal. A bimodal histogram represents a highly unmixed situation like that existing, for example, when half of the box is filled with smoke, which scatters light strongly and results in a high-intensity region of the image, while the other half has no smoke, and, therefore, appears dark on the image and has a relatively low light intensity level. After the sound is initiated, the smoke and air begin to mix. Fewer of the pixels in the image have either a very high or very low value of intensity, whereas increasingly more pixels have a medium intensity value. After 8 s, most of the pixels are at a uniform light intensity, indicating that the system has become well mixed. This is represented by the relatively sharp single peak in the histogram.

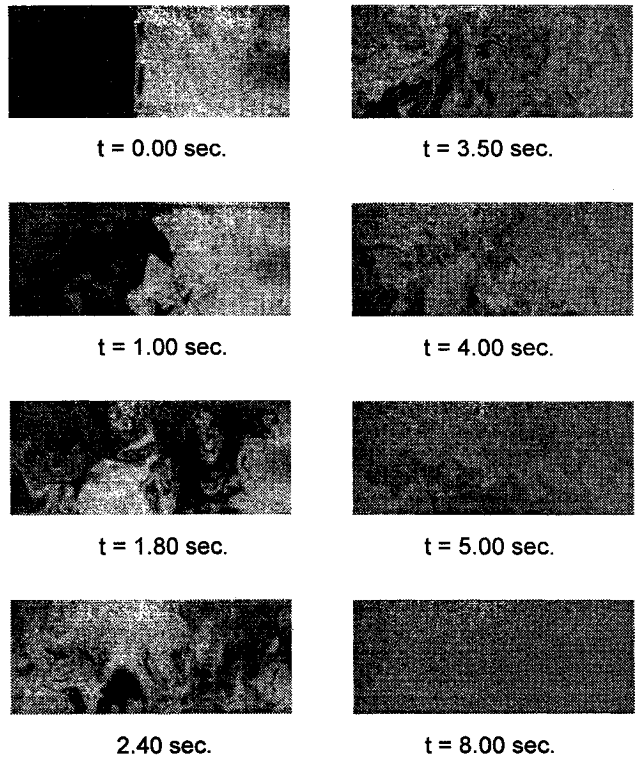

Fig. 2 Sequence of frames showing the mixing process.

The analysis technique used to quantify the mixing observed in this study is based upon a method employed by Liscinsky et al.[13,14] In this technique, a parameter called the spatial unmixedness Us is used to provide a measure of the degree to which a population is mixed. In the present study the definition of Us is somewhat modified from that given by Liscinsky et al.[13,14] Here, Us is defined as

$$Us = \mathrm{Var}(I)/\mathrm{Var}_{t=0}(I) \tag{1}$$

where

$$\mathrm{Var}(I) = \frac{1}{m}\sum_{n=1}^{m}(I_n - I_{\mathrm{av}})^2$$

= spatial variance of pixel intensity

$\mathrm{Var}_{t=0}(I)$ = spatial variance of the $t = 0$ frame

Stated simply, Us is defined here as the variance of the intensity of the frame at time t normalized by the variance of the frame at $t = 0$ (immediately before the sound was initiated). Normalization allows comparisons between runs where the total amounts of smoke added to the box or the intensity of the light sheet may not be exactly the same. The value of Us at $t = 0$ is, from Eq. (1), defined as 1. As the image becomes increasingly mixed, Us approaches 0.

A typical example of the change of Us over time is shown in Fig. 4. In this test, the fundamental longitudinal mode (f = 250 Hz at 144 dB) was driven in the chamber by the two end-mounted drivers operating 180 deg out of phase. Note that local increases (e.g., just after 20 s) in the value of Us do not violate the second law of thermodynamics, because the measured Us represents only a planer slice of the three-dimensional volume. While the overall volume cannot unmix, measured increases in the value of Us represent a localized effect of the three-dimensional mixing process. As the mixing ap-

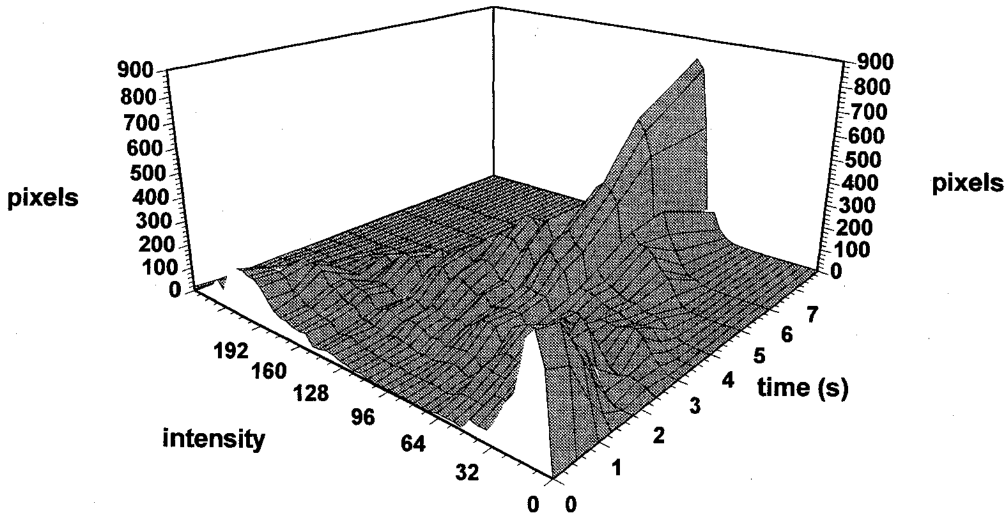

Fig. 3 Evolution of the histograms of the images during mixing.

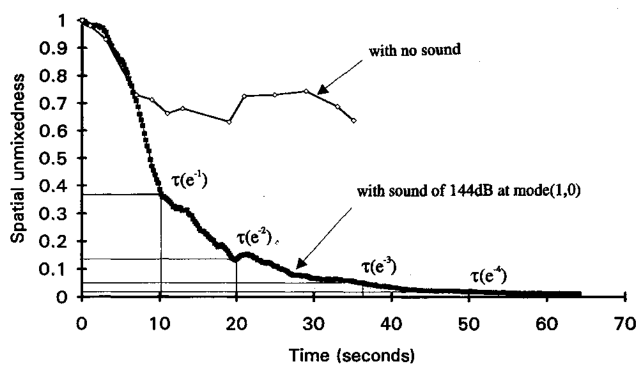

Fig. 4 Spatial unmixedness vs time for a 250-Hz, 144-dB oscillation and without sound.

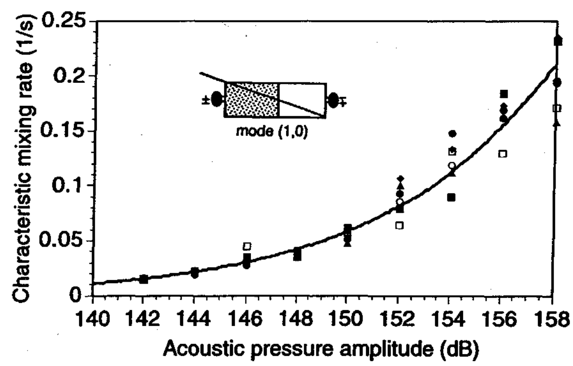

Fig. 5 Characteristic mixing rate vs acoustic pressure amplitude.

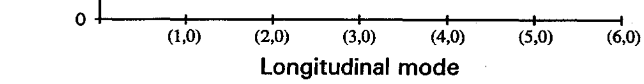

Fig. 6 Dependence of mixing rate upon longitudinal mode at 152 dB.

proaches completion, the length scale of the mixing decreases and the maximum possible fluctuation in *Us* due to three-dimensional effects decreases proportionally. The times at which *Us* has decreased to values of e^{-n} for n = 1, 2, 3, and 4 are shown in Fig. 4. At $\tau(e^{-4})$, *Us* has decreased to 1.83% of its original value. In this study, this was considered to be the point at which the smoke-filled and smoke-free air were fully mixed. The results of a typical measurement of the change of *Us* over time for a test without sound is also shown in Fig. 4 for comparison. In this case, the mixing is driven mainly by convection due to the buoyancy of the smoke (which is slightly warmer than the air temperature), and to a lesser extent, diffusion. Tests without acoustic driving show that while the air in the box rolls around in a slow large-scale motion, mixing at smaller scales proceeds rather slowly. This large-scale motion accounts for the large fluctuations in the value of *Us*, because these motions will carry the cloud of smoke-filled air in such a way that more or less of it is in the plane of the laser sheet at any given time.

Results

Effect of Acoustic Pressure Amplitude on Mixing Rate

A series of experiments were performed to determine the effect of the amplitude of the oscillation upon the characteristic mixing rate (calculated by inverting the measured value of the mixing time) of the smoke and air in the chamber. Because of the chaotic nature of the mixing process, each test was repeated six times to provide better statistical accuracy. Figure 5 shows the measured dependence of the characteristic mixing rate upon the acoustic amplitude. In the tests, drivers mounted on opposite end walls were used to excite the fundamental longitudinal (1,0) mode, as noted by the inset sketch in Fig. 5.

The figure shows that the characteristic mixing rate increases as the amplitude of the acoustic oscillations increases. The curve in the figure is a least-squares fit through the average of the six data at each amplitude of the equation $r = c/p^{\nu}$, where r is the mixing rate, p is the amplitude of the pressure oscillations on a linear scale, and c and ν are determined by curve fitting.

Effect of Acoustic Mode on Mixing Rate

The second series of tests were performed to determine the dependence of the average mixing rate upon the mode of the excited acoustic oscillations. The results of one such series of tests are shown in Fig. 6. In these tests, acoustic oscillations at different longitudinal modes, but at a constant amplitude of 152 dB, were driven using two drivers mounted at opposite ends of the cavity. The drivers were operated 180 deg out of phase for odd modes and in phase for even modes to provide efficient driving. The average mixing rates of the (4,0) and (6,0) modes could not be calculated because Us did not decrease to a value of e^{-4} during the 72 s of storage time available with the camera, indicating that the average mixing rate in both cases is below 0.014 s^{-1}.

In this configuration driving at frequencies corresponding to even acoustic modes appears to produce slower mixing than driving at frequencies corresponding to odd acoustic modes. This can be attributed to the presence of an acoustic velocity antinode at the interface plane between the smoke-filled and smoke-free air for odd modes of oscillation and an acoustic velocity node at this interface plane when even modes are excited. The presence of a large acoustic velocity and the analogous large-amplitude acoustic displacement at the interface appears to promote more rapid mixing in the cavity. Inspection of the measured mixing rates for excitation of odd modes reveals that increasing the mode number while maintaining a constant acoustic pressure amplitude decreases the mixing rate. The same behavior is shown to occur in at least the first two even modes, and although the mixing rates for the (4,0) and (6,0) modes were too low to be measured at this amplitude, the trend was observed to continue at least to the (6,0) mode.

To determine whether the mixing rate depends not only on the mode of acoustic oscillations, but also on the geometric configuration of the acoustic drivers, further tests were conducted using nonsymmetrical driver configurations. The driver placement was found to strongly influence the mixing behavior observed in the chamber. The results of two experiments using different nonsymmetrical driver placements are shown in Fig. 7. In neither of these cases are the mixing rates for even acoustic modes noticeably less than for the odd modes. In fact, in both cases, the mixing provided by the (6,0) mode, which was the slowest mode observed in tests with two, symmetrical, end-mounted drivers, is quite fast with respect to other modes. The marked differences between the results of the three experiments shown in Figs. 6 and 7 demonstrate that the mixing rate of the smoke-filled and smoke-free air in the box depends not only upon the excited acoustic mode, but strongly depends on the placement of the acoustic drivers as well. This result suggests that the mixing is influenced by three-dimensional acoustic processes in the near field of the drivers and/or acoustic streaming phenomena associated with the drivers. These two processes are discussed in the next section.

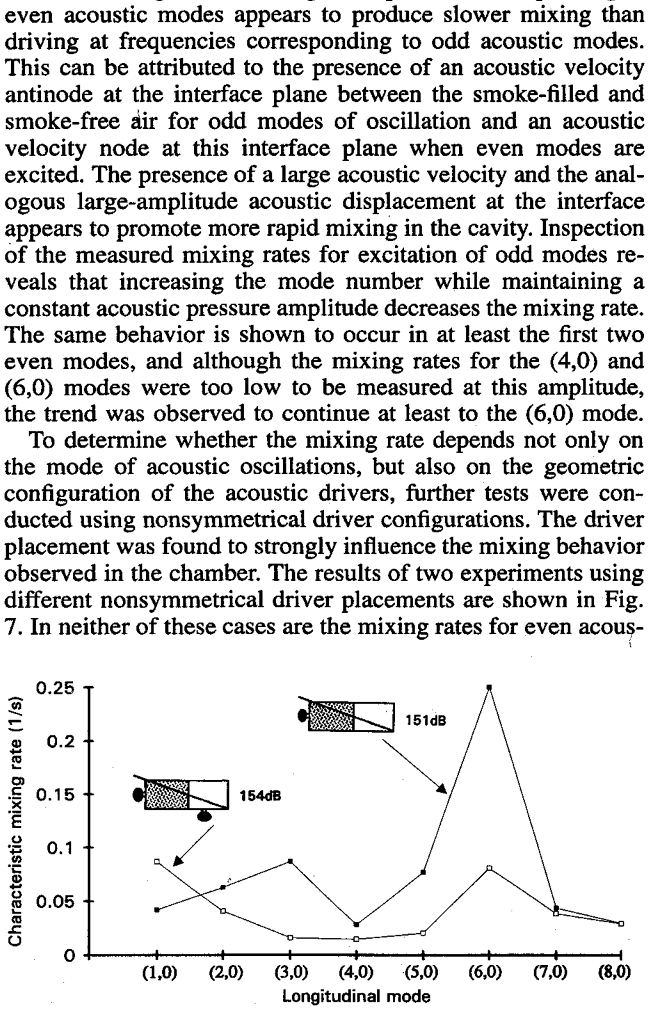

Fig. 7 Comparison of mixing rate dependence upon longitudinal mode for different driving configurations.

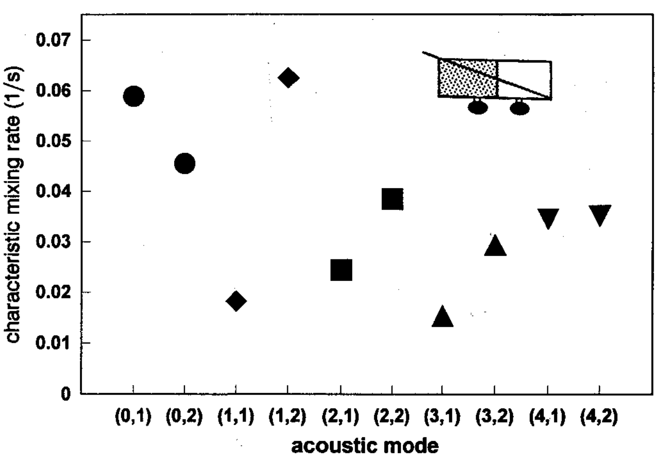

Fig. 8 Dependence of characteristic mixing rate upon mode at 148 dB.

Purely longitudinal modes represent only a fraction of the resonant acoustic modes of the cavity. Therefore, experiments were also performed to investigate the effects of transverse and multidimensional mode excitation upon the mixing rates. Mixing rates measured for a number of transverse and two-dimensional modes are plotted in Fig. 8. The tests were performed using the two bottom-mounted acoustic drivers to excite oscillations of 148 dB amplitude in the chamber. The figure shows that while the second pure transverse mode produces a slower mixing rate than the fundamental transverse mode oscillation, this trend is reversed for two-dimensional modes where driving the (n,2) combined mode results in consistently faster mixing than the (n,1) mode.

Effects of the Drivers

While studying the relationship between the excited acoustic mode and the mixing rate in the experimental test facility, the orientation of the acoustic drivers was found to strongly influence the mixing behavior. To investigate the influence of the driver position upon the average mixing rate, disturbances in the acoustic mode velocity distribution in the vicinity of the driver, and acoustic streaming were investigated with laser Doppler velocimetry (LDV). The acoustic modes in the cavity were determined by assuming solid boundaries with an infinite acoustic impedance. The drivers mounted on the walls present a finite acoustic impedance and acoustic energy enters the volume through the driver openings. This impedance mismatch between the walls and the drivers results in curvature of the standing wave pattern in the vicinity of the drivers. This effect produces variations in the acoustic mode shape due to the position of the drivers. The magnitude of this effect can be seen

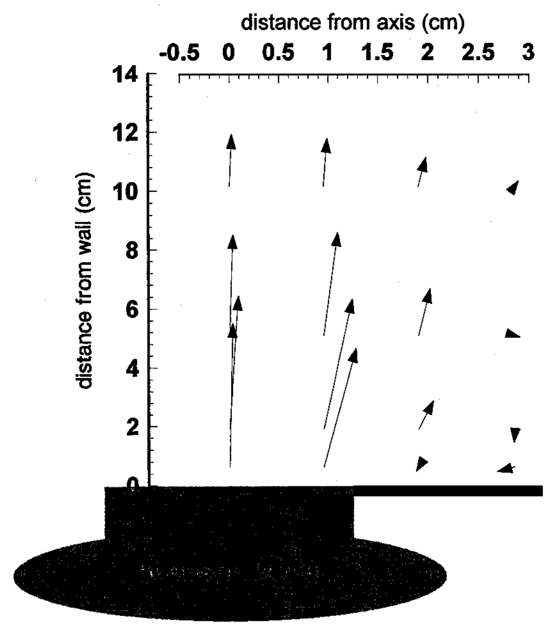

Fig. 9 Acoustic streaming velocity profile in the vicinity of a driver.

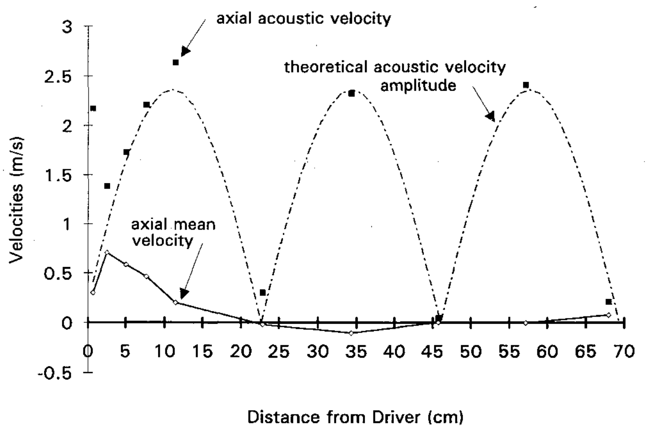

Fig. 10 Dependence of the axial components of the mean and acoustic velocities upon distance from the driver at 150 dB, mode (3,0).

in acoustic velocity measurements. Also, velocity measurements as well as observation of the LDV seeding material (aluminum oxide particles) showed that the oscillating driver induces a mean flow radially inward along the wall and outward along the axis of the driver, as shown in Fig. 9. This acoustic jetting is similar to the streaming patterns observed by Ingard[12] in a study of the impedance of a circular orifice. Figure 10 shows the axial components of the mean and acoustic velocities along the centerline of the cavity for a test in which one end-mounted driver was used to excite the (3,0) mode at 150 dB. The corresponding theoretical velocity amplitude for a standing wave in a solid-walled cavity is shown for comparison. The distortion of the standing wave observed in the vicinity of the driver is due to the impedance mismatch between the acoustic driver and the end wall in which it is mounted. The induced mean jet is shown to extend about half a wavelength from the driver. The flow appears to reverse over the second half wavelength, but the accuracy of the measurements is questionable at such low velocities. The magnitude of the acoustic jetting suggests that it provides an important contribution to the process of mixing in this experimental setup.

Summary and Conclusions

The goal of this study was to investigate the effect of driving high-amplitude, resonant, acoustic oscillations upon the gas phase mixing in a closed cavity. A closed cavity was used so that the effects of the acoustic driving could be isolated from the mixing due to mean flow into and out of the chamber. Experiments were performed in which the mixing rate of smoke-filled and smoke-free air due to excited, resonant, acoustic oscillations were measured in a rectangular cavity with no mean flow. Experimental results show significant enhancement of mixing rate due to the presence of acoustics inside the mixing chamber in comparison to rates measured in the absence of acoustic driving. The rate of gas phase mixing is mainly determined by acoustic pressure amplitude, acoustic mode, and the location of the acoustic drivers on the experimental setup. While the quantitative results obtained in this study are dependent upon the experimental facility, it is believed that the results apply qualitatively to any resonant chamber forced by compact drivers.

The acoustic pressure amplitude has a strong influence on the mixing rate. Increased acoustic pressure amplitudes in the chamber were shown to decrease the mixing time and, thus, increase the mixing rate. The behavior can be approximated by a simple power relationship between mixing rate and acoustic pressure amplitude, given by $r = c/p^{\nu}$, where c and ν are dependent upon the experimental configuration. Whereas the significance of the mixing induced by resonant acoustic driving in an actual combustor, where mean flow is present, will depend on numerous factors, it appears that as the amplitude of the oscillations increases this mechanism will be increasingly important to the total mixing.

The mixing rate was shown to have a complex dependence on the mode of acoustic excitation. When the experimental setup was configured with two acoustic drivers mounted symmetrically at opposite ends of the cavity, the behavior of the mixing appeared to be influenced by the acoustic displacement amplitude at the interface between the smoke-filled and smoke-free air. Acoustic modes with a velocity antinode at the center of the cavity, where the removable partition was located, mixed faster than those that had an acoustic velocity node at the center. Also, in tests with constant acoustic pressure amplitude, the mixing rate was observed to decrease as the frequency of the oscillations increased, which corresponds to a decrease in acoustic displacement amplitude.

The mixing behavior in the chamber was found to depend strongly upon the placement of the acoustic drivers. This is believed to be due to distortions of the acoustic wave patterns in the vicinity of the drivers that results from the impedance mismatch between the drivers and the walls in which they are mounted and acoustic streaming phenomena associated with the drivers. Velocity measurements in the cavity have shown that the velocity of the mean flow jetting induced by the drivers is on the order of the acoustic velocity. This acoustic jetting appears to be responsible for the largest scale motion inside the chamber.

Acknowledgments

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